FRQ 2 Practice

General Notes

- You can use a calculator on this question
- Even if the context of the question only makes sense with whole number, do not round answers unless told to do so. "10.142 eggs" is a perfectly fine answer. Include at least three decimal places. If the answer happens to exact, you do not need to put the extra zeros. e.g. if the answer is 5.7, you do not need to write 5.700. If the answer is 5.216139 thousand eggs, then either "5.216 thousand eggs" or "5216.138 eggs" is correct. But "5216 eggs" is not.

Part A

Part (i) will always ask for a system of equations

- No work needs to be shown.

Part (ii) will always ask for the solution to that system of equations

- It will often be possible to solve the system by hand, but a calculator/Desmos is allowed. Do whatever you find easiest. But spoiler alert: Desmos is easiest.
- No work needs to be shown.
- When using Desmos, make sure to use a tilde (~), not an equal sign. Make sure the variable names match the titles of the columns. Make sure to write W not $W(t)$.

Part B

"Find the average rate of change" is asking you to do slope between two points. You have known slope since Algebra 1.

"Express as a decimal approximation" is not optional. If the answer is $\frac{1}{2}$. You must write 0.5

You must show work. The question will remind you by stating "Show the computations/work that lead to your answer."

For questions asking about whether linear approximation over/underestimates:

- Draw a picture. Label t-values and the names of functions. It is to help you, not the grader. The words you write need to speak for themselves. But a really good picture might win you leniency from the grader.
- Mention concavity of the original model
- Say "secant line" to refer to estimates from the linear approximation
- Mention t-values of secant line
- Mention t-values the question is about
- Mention the relationship between location of secant line and location of original model. Which is on top?

Part C

This question is always about limitations of the model.

If the question is asking you to restrict the domain/range, you never need to bring in outside knowledge. Explicitly mention the relevant information that leads you to restrict domain/range. If the problem is about the sale of Tamagotchis, and the problem says "Tamagotchis were first sold in 1996," be sure in your response to say "Since Tamagotchis were first sold in 1996, we must restrict the domain to..."

If the question is asking about restricting domain/range, be sure to talk about the domain/range of the function, not just the cutoffs of the context. If t is years since 2010 and the model is only applicable until 2020, say "$t \leq 10$" not "$t \leq 2020$"

FRQ #2 Problem A

t	0	3	5
Number of Ashleys (in thousands)	47	50	46

The table above shows the popularity of the name Ashley among newborns in the United States at different times. In the year 1985 ($t = 0$), 47 thousand American newborns were named Ashley. Three years later ($t = 3$), 50 thousand American newborns were named Ashley. Two years after that ($t = 5$), 46 thousand American newborns were given the name Ashley.

The number of American newborns named Ashley can be modelled by the quadratic function N given by $N(t) = at^2 + bt + c$, where $N(t)$ is the number of newborns named Ashley in thousands t years after 1985.

(A) (i) Use the given data to write three equations that can be used to find the values for constants a, b, and c in the expression for $N(t)$.
(ii) Find the values for a, b, and c as decimal approximations.

(B) (i) Use the given data to find the average rate of change of the number of American newborns named Ashley, in thousands per year, from $t = 3$ to $t = 5$ years. Express your answer as a decimal approximation. Show the computations that lead to your answer.
(ii) Use the average rate of change found in (i) to estimate the number of American newborns named Ashley in 1989 ($t = 4$). Show the work that leads to your answer.
(iii) Compare the estimate found in (ii) to the value given by the model, $N(4)$. Using characteristics of the average rate of change and characteristics of the quadratic model, explain why the two estimates differ.

(C) For which t-value, $t = 1$ or $t = 11$, should one have more confidence when using the model N? Give a reason for your answer in the context of the problem.

FRQ #2 Problem B

Mr. Q's home had a small flood. To dry out his walls, he ran a dehumidifier machine. He turned on the machine 4 hours after the flood. So, four hours after the flood ($t = 4$), no water had yet been removed from his walls. Six hours later ($t = 10$), 5 liters of water had been removed from the walls.

The volume of water removed from the walls can be modelled by the function R given by $R(t) = 12 - a(b)^t$, where $R(t)$ is the total volume of water removed t hours after the flood.

(A) (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $R(t)$.
(ii) Find the values for a and b as decimal approximations.

(B) (i) Use the given data to find the average rate of change of the volume of water removed, in liters per hour, from $t = 4$ to $t = 10$ hours. Express your answer as a decimal approximation. Show the computations that lead to your answer.
(ii) Use the average rate of change found in (i) to estimate the volume of water removed 12 hours after the flood. Show the work that leads to your answer.
(iii) The average rate of change found in (i) can be used to estimate the volume of water removed, in liters, after t hours for $t > 12$. Will these estimates, found using the average rate of change, be less than or greater than the volume of water removed predicted by the model R for $t > 12$? Explain your reasoning.

(C) The dehumidifier was not turned on until four hours after the flood. Further, after 11 liters of water have been removed, the dehumidifier will be turned off. Explain how this information can be used to determine the domain limitations for the model R.

FRQ #2 Problem C

The company "Lawn Be Gone" sells pine straw as an alternative for homeowners wishing to replace grass lawns. In 2010 ($t = 0$), the company sold 5000 barrels of pine straw. In 2016 ($t = 6$), the company sold $24{,}000$ barrels of pine straw.

The amount of pine straw sold can be modeled by the exponential function S given by $S(t) = a(b)^t$, where $S(t)$ is the number of barrels sold during year t, and t is the number of years since 2010.

(A) (i) Use the given data to write two equations that can be used to find the values of a and b in the expression for $S(t)$.
(ii) Find the values for a and b as decimal approximations.

(B) (i) Use the given data to find the average rate of change of the quantity sold, in barrels per year, from $t = 0$ to $t = 6$ years. Express your answer as a decimal approximation. Show the computations that lead to your answer.
(ii) Use the average rate of change found in (i) to estimate the quantity of pine straw sold, in barrels, for year $t = 4$. Show the work that leads to your answer.
(iii) The average rate of change found in (i) can be used to estimate the quantity of pine straw sold, in barrels, for year t, where $0 < t < 6$. Over the interval $0 < t < 6$, do the estimates found using the average rate of change overapproximate or underapproximate the quantity predicted by the model S? Explain your reasoning.

(C) Lawn Be Gone first began selling pine straw in 2010. Initially, they only sold to an American audience. Once Lawn Be Gone's yearly sales reached $40{,}000$ barrels, they began shipping pine straw oversees. This dramatically altered the quantity they sold. Explain how this information can be used to determine domain limitations for the model S.

FRQ #2 Problem D

Transistors are a key component of digital electronics. As technology has progressed, transistors have gotten smaller. In the year 1980, the number of transistors that fit on a state-of-the-art integrated circuit was $40{,}000$. By the year 1985, the number of transistors that could fit on an integrated circuit had increased to $100{,}000$.

The number of transistors that fit on an integrated circuit can be modeled by the exponential function N given by $N(t) = ab^{t-1975}$, where $N(t)$ is the number of transistors that fit in the year t.

(A) (i) Use the given data to write two equations that can be used to find the values for the constants a and b in the expression for $N(t)$.
(ii) Find the values for a and b as decimal approximations.

(B) (i) Use the given data to find the average rate of change of the number of transistors, in transistors per year, from $t = 1980$ to $t = 1985$. Show the computation that leads to your answer.
(ii) Interpret the meaning of your answer from (i) in the context of the problem.
(iii) Let $A(t)$ be a model for the number of transistors that fit on an integrated circuit that assumes the number of transistors continues to change linearly at the rate found in (i). This model is different from $N(t)$. If $A(t)$ is used to estimate values for $N(t)$ for $t > 1985$, the error in the estimates will increases as t increases. Explain why this is true.

(C) For which t-value, $t = 1983$ or $t = 1993$, should one have more faith in when using the model N? Give a reason or your answer in the context of the problem.

FRQ #2 Problem E

After the start of the industrial revolution, earth's global average concentration of CO_2 began increasing exponentially. One hundred years after the start of the industrial revolution ($t = 100$), the concentration of CO_2 was 332 ppm. One hundred years after that ($t = 200$), the concentration was 393 ppm.

The global average concentration of CO_2 in earth's atmosphere can be modeled by the exponential function C given by $C(t) = ab^t$, where $C(t)$ is the concentration of CO_2, given in ppm, t years after the industrial revolution began.

(A) (i) Use the given data to write two equations that can be used to find the values for the constants a and b in the expression for $C(t)$.
(ii) Find the values for a and b as decimal approximations.

(B) (i) Use the given data to find the average rate of change of the atmospheric concentration of CO_2, in ppm per year, from $t = 100$ to $t = 200$. Show the computation that leads to your answer.
(ii) Use the average rate of change found in (i) to estimate the global average concentration of CO_2 250 years after the industrial revolution began.
(iii) Rather than use $C(t)$, the average rate of change found in (i) can be used to estimate the concentration of CO_2. Will these estimates, found using the average rate of change, be less than or greater than the CO_2 concentration predicted by the model C during year t for $100 < t < 200$? Explain your reasoning.

(C) Prior to the start of the industrial revolution, earth's average concentration of CO_2 was relatively constant. It did not conform to the exponential trend followed after the industrial revolution began. Find an appropriate lower bound, expressed as a decimal approximation, for the range of the model C. Explain your reasoning.

FRQ #2 Problem F

t (hours)	0	2	5
Mold Concentration ($\frac{\text{spores}}{\text{m}^3}$)	600	560	450

The table gives the mold concentration in the air of a laboratory while a cleaning is conducted. Initially ($t = 0$), there are 600 mold spores per cubic meter of air. Two hours later ($t = 2$), there are 560 spores per cubic meter. Three hours after that ($t = 5$), there are 450 spores per cubic meter.

The concentration of mold in the air can be modelled by the depressed cubic function M given by $M(t) = at^3 + bt + c$, where $M(t)$ represents the number of mold spores per cubic meter of air t hours after the cleaning has begun.

(A) (i) Use the given data to write three equations that can be used to find the values for the constants a, b, and c in the expression for $M(t)$.
(ii) Find the values for a, b, and c as decimal approximations.

(B) (i) Use the given data to find the average rate of change of the concentration of mold, in $\frac{\text{spores}}{\text{m}^3}$ per hour, from $t = 0$ to $t = 2$. Show the computation that leads to your answer.
(ii) Interpret the meaning of your answer from (i) in the context of the problem.
(iii) For $t > 0$, $M(t)$ is decreasing and concave down. The average rate of change found in (i) can be used to estimate the concentration of mold spores for $t > 2$. Will these estimate found using the average rate of change be less than or greater than the concentration of mold spore predicted by M for $t > 2$. Explain your reasoning.

(C) The cleaning lasts until the concentration of mold is 0 spores per cubic meter. Given that M is always decreasing, explain how the range of values of the function M is limited by the context of the problem.

Practice for FRQ 2 Solutions

Problem A - Solutions

(A) (i)

$$47 = a(0)^2 + b(0) + c$$
$$50 = a(3)^2 + b(3) + c$$
$$46 = a(5)^2 + b(5) + c$$

(ii) $a = -0.6$, $b = 2.8$, $c = 47$

How to solve without Desmos:

$$47 = c$$
$$50 = 9a + 3b + c$$
$$46 = 25a + 5b + c$$

$$50 = 9a + 3b + 47$$
$$46 = 25a + 5b + 47$$

$$3 = 9a + 3b$$
$$-1 = 25a + 5b$$

$$b = 1 - 3a$$

$$-1 = 25a + 5(1 - 3a)$$
$$-1 = 25a + 5 - 15a$$
$$-6 = 10a$$
$$a = -0.6$$
$$b = 1 - 3(-0.6) = 2.8$$

(B) (i) $\frac{N(5)-N(3)}{5-3} = \frac{46-50}{5-3} = -2$ thousands of Ashleys per year

(ii) The secant line between the point $(3,50)$ and $(5,46)$ is

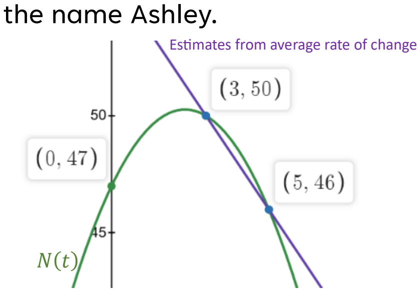
$$y = 46 + (-2)(x - 5)$$

Estimates using the average rate of change are given by

$$N(t) \approx 46 + (-2)(t - 5)$$
$$N(4) \approx 48$$

In 1989 ($t = 4$), there were approximately 48 American newborns given the name Ashley.

(iii) We are comparing outputs from the concave down function $N(t)$ to outputs of the secant line passing through N at $t = 3$ and $t = 5$. Being concave down, the graph of $N(t)$ is above the secant line for $3 < t < 5$. This includes $t = 4$. So, $N(4)$ is greater than the estimate from the average rate of change.



(C) One should have more confidence in using the model $N(t)$ to predict the number of American newborns named Ashley for $t = 1$ than $t = 11$. $t = 1$ is between t-values of data points given ($N(1)$ is interpolation). $t = 11$ is greater than the t-value of any given data point ($N(11)$ is extrapolation), and we have no way of knowing for how many years the model will stay applicable. Interpolation is generally more reliable than extrapolation.

Problem B - Solutions

(A) (i)

$$0 = 12 - a(b)^4$$
$$5 = 12 - a(b)^{10}$$

(ii) $a = 17.189$ and $b = 0.914$

How to solve without Desmos:

$$a(b)^4 = 12$$
$$a(b)^{10} = 7$$

$$b^6 = \frac{7}{12}$$
$$b = \sqrt[6]{\frac{7}{12}} = 0.914$$
$$a = \frac{12}{b^4} = 17.189$$

(B) (i) $\frac{R(10)-R(4)}{10-4} = \frac{5-0}{10-4} = 0.833$

The average rate of change is 0.833 liters per hour

(ii) The secant line between the point $(4,0)$ and $(10,5)$ is

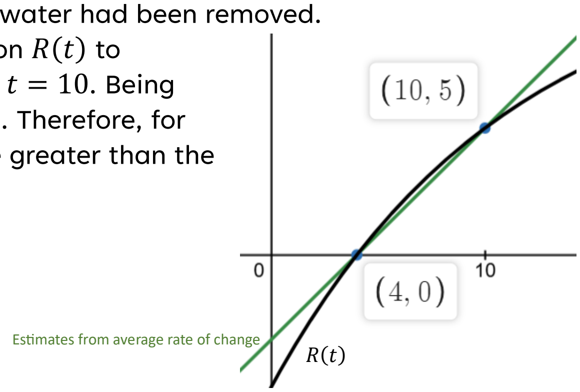
$$y = 5 + (0.833)(x - 10)$$

Estimates using the average rate of change are given by

$$R(t) \approx 5 + (0.833)(t - 10)$$
$$R(12) \approx 6.667$$

By 12 hours after the flood, approximately 6.667 liters of water had been removed.

(iii) We are comparing outputs from the concave down function $R(t)$ to outputs of the secant line passing through R at $t = 4$ and $t = 10$. Being concave down, $R(t)$ is less than the secant line for $t > 10$. Therefore, for $t > 12$ the estimates from the average rate of change are greater than the predictions using R.



(C) The model makes predictions for the effect of the dehumidifier for $t < 4$. But in reality, since the dehumidifier was not turned on until four hours after the flood, the dehumidifier had no effect during this time. So we exclude $t < 4$ from the domain. Setting $R(t) = 11$, yields $t = 31.661$. After this t-value, the model has outputs greater than 11. But in reality, the dehumidifier is turned off at $t = 31.661$ upon having removed 11 liters of water. So the function modeling the effect of the dehumidifier has no meaning after $t = 31.661$. The domain of $R(t)$ is $4 \leq t \leq 31.661$.

Problem C - Solutions

(A) (i)

$$5000 = ab^0$$
$$24000 = ab^6$$

(ii) $a = 5000$ and $b = 1.299$
How to solve without Desmos:

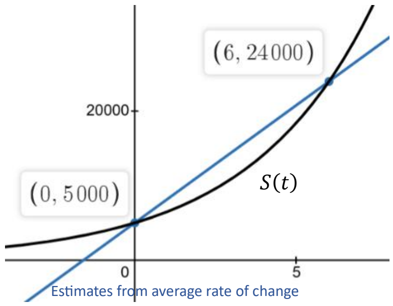
$$5000 = a$$
$$24000 = ab^6$$

$$24000 = 5000b^6$$
$$\frac{24}{5} = b^6$$
$$b = \sqrt[6]{\frac{24}{5}} = 1.299$$

(B) (i) $\frac{S(6)-S(0)}{6-0} = \frac{24000-5000}{6-0} = 3166.667$
The average rate of change is 3166.667 barrels per year.

(ii) The secant line between the point $(0{,}5000)$ and $(6{,}24000)$ is
$$y = 24000 + (3166.667)(x - 6)$$
Estimates using the average rate of change are given by
$$S(t) \approx 24000 + (3166.667)(t - 6)$$
$$S(4) \approx 24000 + (3166.667)(4 - 6)$$
$$S(4) \approx 17666.667$$
In the year $t = 4$, approximately $17{,}666.667$ barrels of pine straw are sold.

(iii) We are comparing outputs from the concave up function $S(t)$ to outputs of the secant line passing through S at $t = 0$ and $t = 6$. Between $t = 0$ and $t = 6$, the secant line has the greater output since S is concave up. So, for $0 < t < 6$, the estimates found using the average rate of change overapproximate the quantities predicted by $S(t)$.



(C) For $t < 0$, the model has nonzero output values. But in reality, Lawn Be Gone did not sell prior to 2010 ($t = 0$). So, we must exclude negative t-values from the domain. Solving $S(t) = 40000$ and finding $t = 7.954$, we find when the sales trend is “dramatically altered” and the model loses meaning. The domain of S is $0 \leq t \leq 7.954$.

Problem D - Solutions

(A) (i)

$$40000 = ab^{1980-1975}$$
$$100000 = ab^{1985-1975}$$

(ii) $a = 16000$ and $b = 1.201$

How to solve without Desmos:

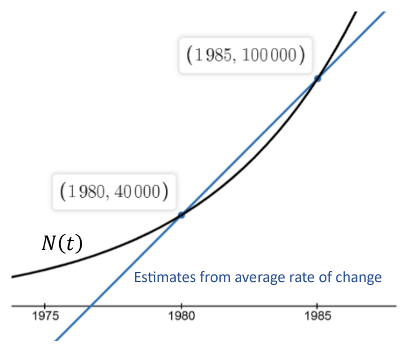
$$a = \frac{40000}{b^5}$$
$$100000 = \left(\frac{40000}{b^5}\right) b^{10}$$
$$100000 = 40000b^5$$
$$\frac{5}{2} = b^5$$
$$b = \sqrt[5]{\frac{5}{2}} = 1.201124$$
$$a = \frac{40000}{b^5} = \frac{40000}{(1.201124)^5} = 16000$$

(B) (i) $\frac{N(1985)-N(1980)}{1985-1980} = 12000$

The average rate of change is 12,000 transistors per year.

(ii) From 1980 to 1985, the number of transistors that fit on an integrated circuit on average increased by 12,000 transistors per year.

(iii) We are comparing outputs from the concave up function $N(t)$ to outputs of $A(t)$, the secant line passing through N at $t = 1980$ and $t = 1985$. Being concave up, $N(t)$ is greater than the secant line $A(t)$ for $t > 1985$. Being concave up, the rate of change of $N(t)$ is always increasing, so it will grow even further past $A(t)$ as the secant line's rate of change is constant. The error, which is the difference between the estimate from $A(t)$ and the value of $N(t)$, is increasing as t increases.



(C) One should have more confidence in using the model $N(t)$ to predict the number of transistors for $t = 1983$ than for $t = 1993$. There is insufficient data to know for how many years the model will stay applicable. Regression models are typically more reliable for interpolation than extrapolation.

Problem E - Solutions

(A) (i)

$$ab^{100} = 332$$
$$ab^{200} = 393$$

(ii) $a = 280.468$ and $b = 1.002$

How to solve without Desmos:

$$\frac{ab^{200}}{ab^{100}} = \frac{393}{332}$$
$$b^{100} = \frac{393}{332}$$
$$b = \sqrt[100]{\frac{393}{332}} = 1.001688$$
$$a = \frac{332}{b^{100}} = 280.468193$$

(B) (i) $\frac{C(200)-C(100)}{200-100} = \frac{393-332}{200-100} = 0.61$

The average rate of change is 0.61 ppm per year.

(ii) The secant line between the point $(100{,}332)$ and $(200{,}393)$ is

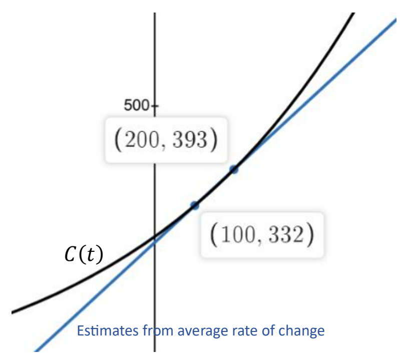
$$y = 393 + 0.61(x - 200)$$

Estimates using the average rate of change are given by

$$C(t) \approx 393 + 0.61(t - 200)$$
$$C(250) \approx 423.5$$

250 years after the industrial revolution began, the global average concentration of CO_2 was approximately 423.5 ppm.

(iii) We are comparing outputs of the concave up function C with outputs of the secant line that passes through C at $t = 100$ and $t = 200$. Because C is concave up over its entire domain, the secant line is above the graph of C for $100 < t < 200$. The estimate found for times in $100 < t < 200$ using the average rate of change are greater than the CO_2 concentration levels predicted by the model C.



(C) Since the exponential trend does not apply prior to the start of the industrial revolution, the model should not be applied to t values less than zero. Because C is an increasing function (exponential growth), its minimum value occurs at the start of its domain. The range of $C(t)$ has lower bound $C(0) = 280.468$.

Problem F - Solutions

(A) (i)

$$600 = a(0)^3 + b(0) + c$$
$$560 = a(2)^3 + b(2) + c$$
$$450 = a(5)^3 + b(5) + c$$

(ii) $a = -0.476$, $b = -18.095$, $c = 600$

How to solve without Desmos:

$$600 = c$$
$$-40 = 8a + 2b$$
$$-150 = 125a + 5b$$

$$b = -20 - 4a$$
$$-150 = 125a + 5(-20 - 4a)$$

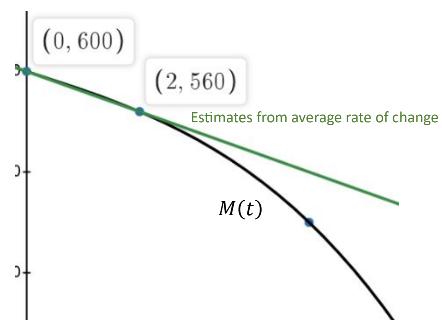
$$-50 = 105a$$
$$a = -\frac{50}{105} = -0.47619$$
$$b = -20 - 4a = -18.095238$$

(B) (i) $\frac{M(2)-M(0)}{2-0} = \frac{560-600}{2-0} = -20$

The average rate of change is $-20\,\frac{\text{spores}}{\text{m}^3}$ per hour

(ii) Over the first two hours after cleaning has begun, the concentration of mold in the air of laboratory decreases at an average rate of $20\,\frac{\text{spores}}{\text{m}^3}$ per hour.

(iii) We are comparing outputs of $M(t)$ to outputs of the secant line passing through M at $t = 0$ and $t = 2$. Since M is concave down for $t > 0$, M is concave down for $t > 2$. Since M is concave down, it falls below the secant line for $t > 2$. Therefore, the estimates found using the average rate of change are greater than the concentrations predicted by M for $t > 2$.



(C) Because M is always decreasing, its maximum value occurs at the start of the domain. The maximum of M is $M(0) = 600$. From this output value, M continues to decrease. Once M reaches a value of zero, the cleaning stops and the output of M no longer is meaningful. So, the minimum of M must be zero. The range is $0 \le M(t) \le 600$.